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Final Technical Report

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Principle Investigator
D. A. Calahan
Department of Electrical and Computer Engineering
University of Michigan
Ann Arbor, Michigan

Part I: NUMERICAL STUDIES IN / COMPUTER AIDED DESIGN

Sponsored by

Directorate of Mathematical and Information Sciences Air Force Office of Scientific Research (4/1/75 - 3/31/80)

Part II: STUDIES IN HIGH SPEED COMPUTATION

Sponsored by

Air Force Flight Dynamics Laboratory Wright-Patterson Air Force Base (4/1/78 - 3/31/80)



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## I. Report Organization

Corresponding to the two Air Force agencies that supported this research, the report is presented in two parts.

- (1) PART I: Numerical Studies in Computer-Aided Design.

  Sponsored by AFOSR, this parent study involved general research in the mathematical modeling of vector processors and the development of vectorized sparse matrix procedures.
- (2) PART II: Studies in High Speed Computation. Funded by AFFDL in the fourth and fifth years of the parent study, this research involved a more detailed study of the CRAY-1 processor and its application to simulation of aerodynamic fluid flow.

#### II. Summary of Research

This grant was originally concerned solely with the development of algorithms and the evaluation of algorithmic complexity for the direct (contrast iterative) solution of large sets of sparse simultaneous equations on vector processors. This analytical study was preceded by benchmarks of a number of commercial processors. The speedups observed in these benchmark studies, especially for the CRAY-1, suggested the subsequent CRAY-1 benchmark study on 2-D and 3-D aerodynamic fluid flow. These two studies found common ground in development of a CRAY-1 logical/timing simulator, which greatly facilitated both algorithm development and the coding of benchmarks.

The principal accomplishments of this grant are summarized in the following.

PART I

## A. Benchmarks

In 1976, a report [27] was prepared on the benchmarking and mathematical modeling of a number of early commercial vector processors (Texas Instruments ASC, Control Data STAR 100, and Cray Research CRAY-1). This was the earliest public benchmark of these processors and, even though it involved a simple linear equation solver, several hundred copies of the benchmark report were requested. The results are summarized in Figure 1.

## B. Software for sparse solution

In 1977, a Fortran-coded vectorized version of a well-known scalar sparse equation solution algorithm [29] was developed. This report anticipated by several years the exploitation of vector processing for the solution of sparse problems. Implemented in Fortran, however, it could not utilize the particular data flow characteristics of a memory hierarchial machine such as the CRAY-1. Indeed, its maximum processing speed is limited to approximately 1/4 the maximum speed of the CRAY-1, in spite of its vectorized formulation [1][4].

In 1979, an assembly-coded block-oriented sparse solver was developed [4][5] to exploit the data flow of the CRAY-1. It was found that, since the CRAY-1 required blocking of even dense matrices due to its memory hierarchy, the same general sparse solver could be used to solve - with little overhead for generality but with high efficiency from assembly coding - banded, blocked tridiagonal, and full matrices. This appears to represent a new application of sparse solvers, i.e., the replacement of a number of assembly codes written for specific sparsity structures.

## C. Complexity of vectorized sparse solution

In 1975-6 [10][28], the concept of average vector lengh was proposed as a useful representation of the vectorizability of the solution of finite element grids. Complexity formula and timing estimates were given for solution of grids. Example results are given in Figure 2.

# D. Equation ordering for vector solution.

In 1979 [1][2], it was observed that, in the solution of sparse equations, vectors resulted either from exploiting local density of a sparse matrix or a global pattern associated with the problem structure (or, equivalently, the associated graph of the matrix). It was shown that certain symmetry-exploiting operations on the graph, such as folding and rotation, yielded an equation ordering which resulted in vector operations in the solution of the equations. For example, the reordering of the matrix of Figure 3 yields the "striped" structure of Figure 4, which is shown in [2] to be more amenable to vectorized solution.

#### PARTS I and II

## A. CRAY-1 Simulator

In 1978, a CRAY-1 logical and clock-level timing simulator and a cross assembler were implemented for the Amdahl 470. This program was subsequently converted to the CRAY-1 by Los Alamos Scientific Laboratory and Lawrence Livermore Laboratory to study critical high performance algorithms which even the CRAY-1, without an interrupt capability, cannot itself monitor.

## B. Equation Solving Codes

A number of high performance codes were developed with the aid of the CRAY-1 simulator. Among other notable results, it was shown that vector accumulation loops, encountered in nearly all linear algebra codes, could be more efficiently implemented by avoiding functional unit chaining, supposedly a feature of the CRAY-1 designed to produce concurrent operation of functional units. These codes have been documented in [20], and have been sent to Cray Research, to Bell Telephone Laboratories, and to Los Alamos Scientific Laboratory, on their request.

#### PART II

### A. Aerodynamic Fluid Flow

In 1978 and 1979, kernels of an explicit Navier Stokes code developed at AFFDL were coded in both CRAY-1 assembly language and CRAY-1 Fortran. Results are reported in [3] and [6], joint papers with AFFDL. A summary of the speedups obtained viz a viz the CDC 6600 and 7600 are given in Table 1 below.

COMPUTER	CODE	RATIO
CYBER 74	Scalar	1.0
CDC 7600	Scalar	5.2
CRAY-1	Scalar	15.7
CRAY-1	Vector	127.7
CRAY-1	Assembly	144.2

Table 1. Relative execution rates of Computers in explicit solution.

## III. Coupling Activities

- A. Seminars on vector processing
  - 1. Two seminars at Los Alamos Scientific Laboratory
  - 2. One seminar at the University of Minnesota.
  - 3. One seminar at AFFDL.

## B. Visits

- 1. One visit to AFWL.
- 2. Two visits to AFFDL, prior to institution of funding.

## C. Industrial Consulting

- With General Electric and EPRI, on the vector analysis
  of electric power system grids, resulting in a report
  [30].
- 2. With Mobil Research and Development (Dallas) on the study of vectorization of 3-D diffusion codes associated with oil reservoir drilling and management.
- 3. With Bell Telephone Laboratories, to study the solution of sparse equations representing communication systems.

#### D. Other

- A one-week short course at the University of Michigan on High Speed Computation was taught in 1977, 1978, and 1979.
   Among 41 attendees in 1979 were representatives of Air Force Headquarters (Pentagon), AFWL, RADC, and AFFDL, as well as NRL and Picatinney Arsenal.
- Visiting Research Scientist at the Los Alamos Scientific
   Laboratory (1979- ) to evaluate particle physics codes.

 Visiting scientist at AFFDL (1979- ) to assist in development of vectorized explicit/implicit fluids codes.

### Grant-Supported Publications

### Journal Publications

[ 1]. (Invited) Calahan, D. A., "Vector Processors: Models and Applications," IEEE Trans. on Circuits and Systems, vol. CAS-26, No. 9, September 1979, pp.715-726.

## Conference Publications

- [2]. Calahan, D. A., "Vectorized Sparse Elimination," Proc. Scientific Computer Information Exchange Meeting, Livermore, CA, September 12-13, 1979, pp. 103-114.
- [ 3]. Shang, J., P. G. Buning, W. L. Hankey, M. C. Wirth, D. A. Calahan, and W. Ames, "Numerical Solution of the 3-D Navier Stokes Equation on the CRAY-1 Computer," Proc. Scientific Computer Information Exchange Meeting, Livermore, CA., September 12-13, 1979, pp. 159-166.
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- [14]. Calahan, D. A., "An Evaluation of the CRAY-1 as an Aerodynamic Simulation Processor," AIAA Conference on Plasma and Fluid Dynamics, Williamsburg, VA., June, 1979.
- [15]. Buning, P., "Preliminary Report on the Evaluation of the CRAY-1 as a Numerical Simulation Processor," 11th AIAA Conf. on Plasma and Fluid Dynamics, Seattle, July, 1978.
- [16]. Calahan, D. A., "Comments on Current Vector Processors," Workshop on Current Vector Processors, NASA/Langley, October, 1976.
- [17]. Calahan, D. A., "Comparison of Current Vector Processors,"
  Proc. Third ICASE Conf. on Scientific Computing, Williamsburg,
  Va., April, 1976.
- [18]. Calahan, D. A., et al, "Benchmarks for Vector Processors,"
  Third ICASE Conference on Scientific Computing, NASA/
  Langley Research Center, April, 1976.
- [19]. Calahan, D. A., "Complexity Analysis of Matrix Benchmarks on the CRAY-1," Symposium on New Directions and Recent Results in Algorithms and Complexity, Carnegie-Mellon University, April, 1976.

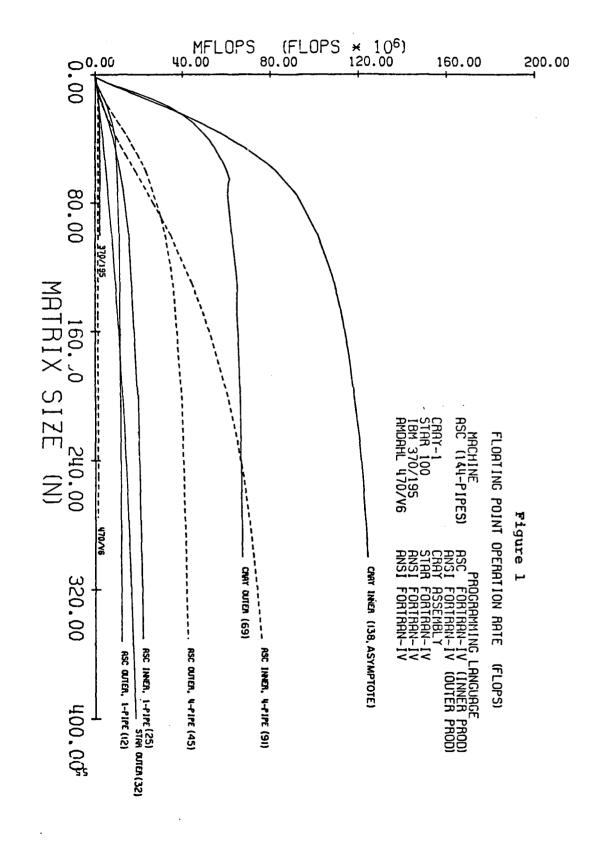
### Reports

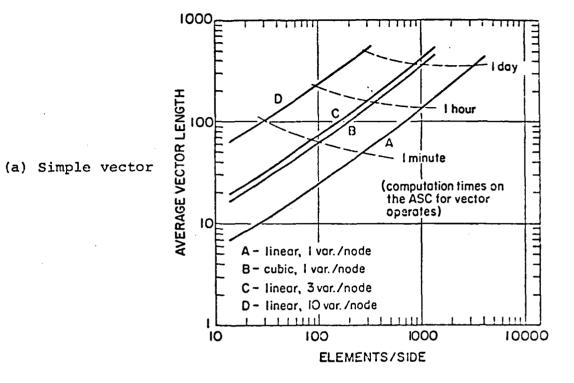
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- [30]. Barry, D. E., C. Pottle, and K. A. Wirgau, "Technology Assessment Study of Near Term Computer Capabilities and Their Impact on Power Flow and Stability Simulation Programs," EPRI Report EL-946, General Electric Co., Schenectady, December, 1978.





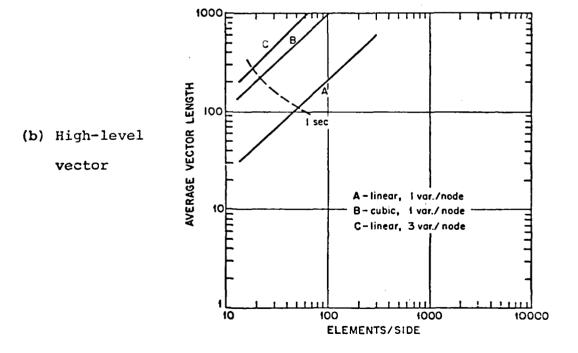
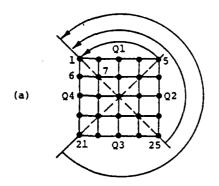
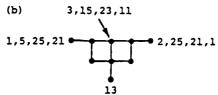
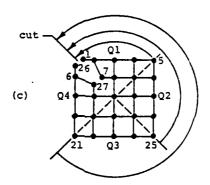


Figure 2. Average vector lengths for direct solution of finite element grids, with different definitions of a vector primitive.







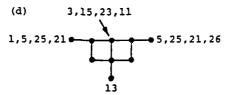


Figure 3. Connectivity graph of finite difference difference grid; notation of quadrants (a) into single quadrant representation (b); rotation with cut and creation of nodes ((b)-(c)).

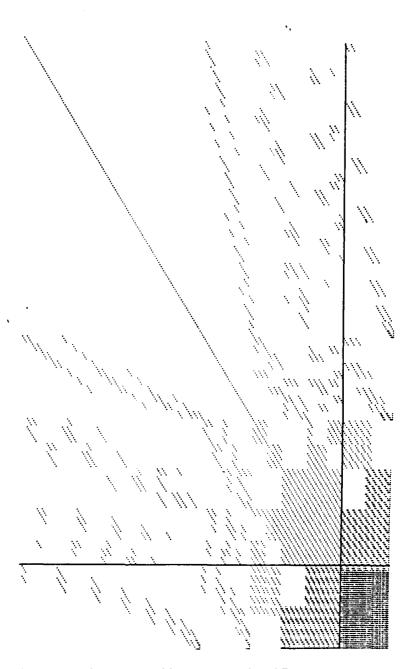


Figure 4. Matrix of a dissected 17x17 5-point finite difference grid; after rotation

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	This report summarizes research investigating (1) algorithms for the vector-ized direct solution of sparse simultaneous equations, and (2) the application of vector processors to aerodynamic fluid flow. The research was performed between 4/1/75 and 3/31/80.			